Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Friday 5 June 2015 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P44840A

1. Particles P and Q move in a plane with constant velocities. At time $t = 0$ the position vectors of P and Q, relative to a fixed point O in the plane, are $(16\mathbf{i} - 12\mathbf{j})$ m and $-5\mathbf{i} + 4\mathbf{j})$ m respectively. The velocity of P is $(\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of Q is $(2\mathbf{i} + \mathbf{j})$ m s^{-1}.

Find the shortest distance between P and Q in the subsequent motion.

(7)

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2. When a woman walks due North at a constant speed of 4 km h^{-1}, the wind appears to be blowing from due East. When she runs due South at a constant speed of 8 km h^{-1}, the speed of the wind appears to be 20 km h^{-1}.

Assuming that the velocity of the wind relative to the earth is constant, find

(i) the speed of the wind,

(ii) the direction from which the wind is blowing.

(6)

3.

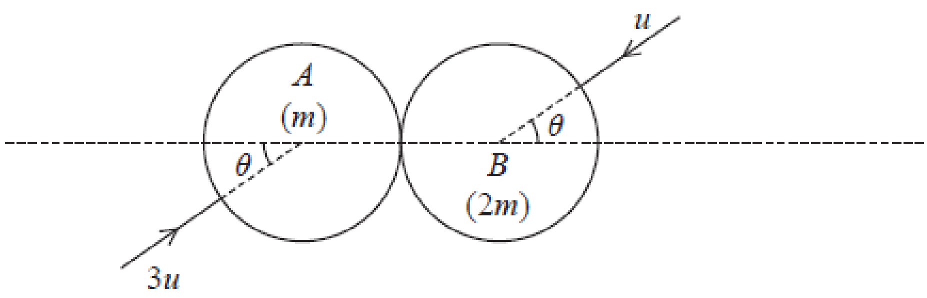

Figure 1

Two smooth uniform spheres A and B with equal radii have masses m and $2m$ respectively. The spheres are moving in opposite directions on a smooth horizontal surface and collide obliquely. Immediately before the collision, A has speed $3u$ with its direction of motion at an angle θ to the line of centres, and B has speed u with its direction of motion at an angle θ to the line of centres, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{1}{8}$.

Immediately after the collision, the speed of A is twice the speed of B.

Find the size of the angle θ.

(12)

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4. A car of mass 900 kg is moving along a straight horizontal road with the engine of the car working at a constant rate of 22.5 kW. At time t seconds, the speed of the car is v m s^{-1} $(0 < v < 30)$ and the total resistance to the motion of the car has magnitude $25v$ newtons.

(*a*) Show that when the speed of the car is v m s^{-1}, the acceleration of the car is

$$\frac{900 - v^2}{36v} \text{ m s}^{-2}.$$

(3)

The time taken for the car to accelerate from 10 m s^{-1} to 20 m s^{-1} is T seconds.

(*b*) Show that

$$T = 18 \ln \frac{8}{5}.$$

(5)

(c) Find the distance travelled by the car as it accelerates from 10 m s^{-1} to 20 m s^{-1}.

(6)

5.

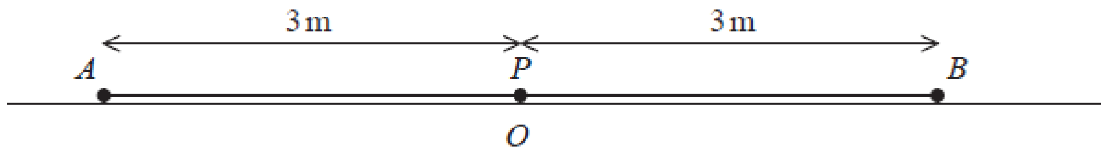


Figure 2

A particle P of mass 1.5 kg is attached to the midpoint of a light elastic spring AB, of natural length 2 m and modulus of elasticity 12 N. The end A of the spring is attached to a fixed point on a smooth horizontal floor. The end B is held at a point on the floor where AB = 6 m.

At time $t = 0$, P is at rest on the floor at the point O, where AO = 3m, as shown in Figure 2. The end B is now moved along the floor in such a way that AOB remains a straight line and at time t seconds, $t \geq 0$,

$$AB = \left(6 + \frac{1}{4}\sin 2t\right) \text{ m.}$$

At time t seconds, $AP = (3 + x)$ m.

(a) Show that, for $t \geq 0$,

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 16x = 2\sin 2t.$$

(5)

The general solution of this differential equation is

$$x = C\cos 4t + D\sin 4t + \frac{1}{6}\sin 2t,$$

where C and D are constants.

(b) Find the time at which P first comes to instantaneous rest.

(5)

6.

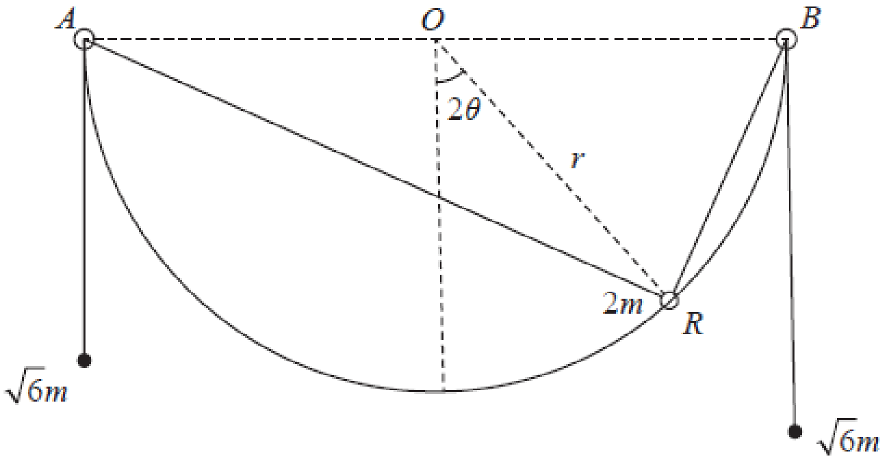


Figure 3

A smooth wire, with ends A and B, is in the shape of a semicircle of radius r. The line AB is horizontal and the midpoint of AB is O. The wire is fixed in a vertical plane. A small ring R of mass $2m$ is threaded on the wire and is attached to two light inextensible strings. One string passes through a small smooth ring fixed at A and is attached to a particle of mass $\sqrt{6}m$. The other string passes through a small smooth ring fixed at B and is attached to a second particle of mass $\sqrt{6}m$. The particles hang freely under gravity, as shown in Figure 3. The angle between the radius OR and the downward vertical is 2θ, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$.

(*a*) Show that the potential energy of the system is

$$2mgr(2\sqrt{3}\cos\theta - \cos 2\theta) + \text{constant.}$$

(6)

(*b*) Find the values of θ for which the system is in equilibrium.

(4)

(*c*) Determine the stability of the position of equilibrium for which $\theta > 0$.

(3)

7.

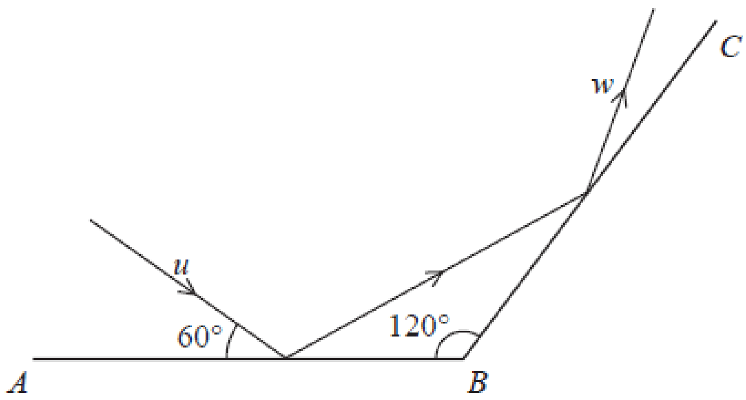


Figure 4

Figure 4 represents the plan view of part of a smooth horizontal floor, where AB and BC are smooth vertical walls. The angle between AB and BC is 120°. A ball is projected along the floor towards AB with speed u m s^{-1} on a path at an angle of 60° to AB. The ball hits AB and then hits BC. The ball is modelled as a particle. The coefficient of restitution between the ball and each wall is $\frac{1}{2}$.

(*a*) Show that the speed of the ball immediately after it has hit AB is $\dfrac{\sqrt{7}}{4}u$.

(6)

The speed of the ball immediately after it has hit BC is w m s^{-1}.

(*b*) Find w in terms of u.

(7)

TOTAL FOR PAPER: 75 MARKS

END

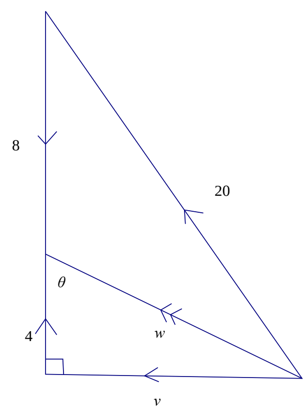
June 2015

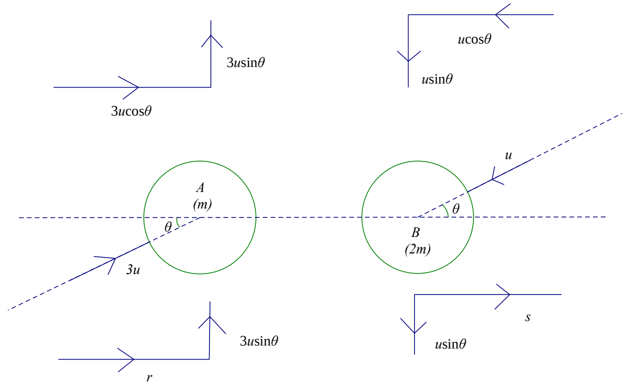
6680 Mechanics 4
Mark Scheme

Question Number	Scheme	Marks	Notes
1	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Find position vector of one particle relative to the other . $\mathbf{r}_P = \begin{pmatrix} 16+t \\ -12+2t \end{pmatrix}$, $\mathbf{r}_Q = \begin{pmatrix} -5+2t \\ 4+t \end{pmatrix}$
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	$d^2 = (21-t)^2 + (-16+t)^2$	M1	Pythagoras
	$\frac{\mathrm{d}}{\mathrm{d}t} d^2 = -2(21-t) + 2(-16+t)\ (= -74 + 4t)$	M1	Differentiate d or d^2 wrt t
			Set derivative = 0 and solve for t
	Min when $t = 18.5\,(\mathrm{s})$	A1	
	Relative position $\begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix}$, distance $\sqrt{2.5^2 + 2.5^2}$ (m)	M1	Substitute their t to find d
	$= \sqrt{\frac{25}{2}} = 3.54$ (m)	A1	
		[7]	
	See over for alternatives.		

Question Number	Scheme	Marks	Notes
alt1	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Position of P relative to Q
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	$d^2 = (21-t)^2 + (-16+t)^2 \ \left(= 2t^2 - 74t + 697\right)$	M1	Use Pythagoras to express d^2 as a quadratic in t
		M1	Complete the square
	$2(t-18.5)^2(-684.5+697)$	A1	Correct as far as $2(t-18.5)^2 + ...$
	Min $d^2 = 697 - 684.5$	M1	Use completed square to find minimum value for their expression
	Min. $d = \sqrt{697-684.5} = \sqrt{12.5}$	A1	
alt2	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Position of P relative to Q
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	Relative velocity $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	M1	
	$: \begin{pmatrix} 21-t \\ -16+t \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -(21-t) + (-16+t) = 0,$	M1	Set scalar product of relative position and relative velocity = 0 and solve for t.
	$t = 18.5\,(\mathrm{s})$	A1	
	Relative position $\begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix}$, distance $\sqrt{2.5^2 + 2.5^2}$ (m)	M1	Substitute their t to find d
	$= \sqrt{\frac{25}{2}} = 3.54$ (m)	A1	
	See over for alternative		

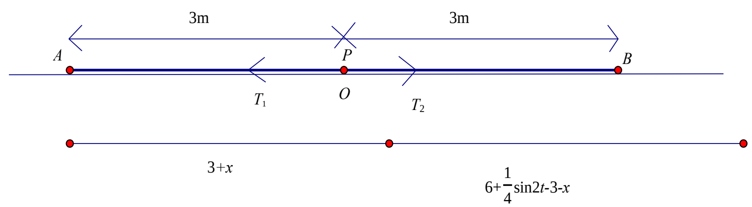
Question Number	Scheme	Marks	Notes
alt 3	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Initial position of P relative to Q
	$= \begin{pmatrix} 21 \\ -16 \end{pmatrix}$	A1	Accept +/-
	Relative velocity $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	M1	
		M1	Use scalar product to find $\cos\theta$
	$\cos\theta = \dfrac{-37}{\sqrt{2}\sqrt{697}}$ (-0.998....)	A1	Accept +/-
	$d = PQ\sin\theta$	M1	Use trig to find distance
	$= \sqrt{697} \times \sqrt{1 - \dfrac{37^2}{2\times 697}} = \dfrac{5}{\sqrt{2}} \approx 3.54$	A1	
		[7]	

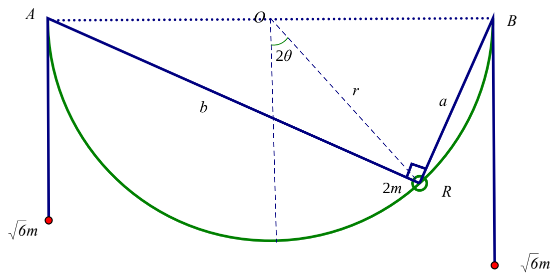
Question Number	Scheme	Marks	Notes
2		B1	Either triangle of velocities
		M1	Two triangles combined using their common velocity
		A1	Correct diagram seen or implied
	Correct method to obtain one of v, w, θ	DM1	($v = 16, w = 16.5, \theta = 76°$) Dependent on the previous M1
	speed is 16.5(km h^{-1})	A1	$4\sqrt{17}$
	Direction S $76°$ E or equivalent	A1	$104°$ or equivalent
		[6]	
Alt	Velocity of wind = w		
	$w = -v\mathbf{i} + 4\mathbf{j}$	B1	one correct equation
	$w = a\mathbf{i} + b\mathbf{j} - 8\mathbf{j}$ $\quad a^2 + b^2 = 400$	M1	2nd equation and compare coefficients
	coeff $\mathbf{j}$: $\quad 4 = b - 8 \quad b = 12$	A1	2 correct eqns
	$\mathbf{i}$: $\quad -v = a$		
	$a^2 + 144 = 400 \Rightarrow a = -16 \quad (v > 0)$	DM1	Dependent on the previous M1
	$\lvert w \rvert = \sqrt{4^2 + 16^2} = 4\sqrt{17}$	A1	
	Bearing $104°$	A1	or equivalent

Question Number	Scheme	Marks	Notes
3		B1	After collision $u\sin\theta$ and $3u\sin\theta$ perpendicular to l of c Seen or implied
	CLM : $r+2s=3u\cos\theta-2u\cos\theta\left(=u\cos\theta\right)$	M1	Requires all four terms but condone sign errors and consistent sin/cos confusion.. Must be dimensionally consistent
		A1	Correct unsimplified equation
	Impact: $s-r=e\times 4u\cos\theta\left(=\dfrac{u\cos\theta}{2}\right)$	M1	Must be the right way round, but condone sign errors and consistent sin/cos confusion
		A1	Correct unsimplified equation. Signs consistent with CLM equation.
	$\Rightarrow r=0, s=\dfrac{u\cos\theta}{2}$	DM1	Solve the simultaneous equations to find the horizontal components of velocities. Dependent on the two preceding M marks
		A1	Both correct
	After the collision: $(3u\sin\theta)^2+r^2=4\left((u\sin\theta)^2+s^2\right)$	M1	Use $v_A=2v_B$. Condone 2 in place of 4.
		A1ft	Correct unsimplified equation (in r and s)
	$9u^2\sin^2\theta=4u^2\sin^2\theta+4.\dfrac{u^2}{4}\cos^2\theta$	A1	Obtain an equation in θ (correct only)
	$\tan^2\theta=\dfrac{1}{5}$, $\theta=24.1(^\circ)$ (0.421 radians)	DM1	Solve for θ. Dependent on the previous M1
		A1	Correct to 3 sf or better

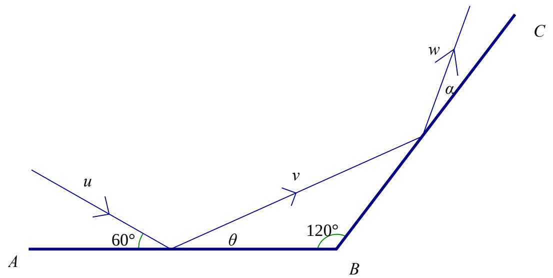
Question Number	Scheme	Marks	Notes
3 alt	For those who prefer everything with trig:		
	$v_A \sin\alpha = 3u\sin\theta$, $v_B \sin\beta = u\sin\theta$	B1	Perpendicular to the l.o.c.
	$m.3u\cos\theta - 2m.u\cos\theta = mv_A\cos\alpha + 2mv_B\cos\beta$	M1	CLM
	$(u\cos\theta = v_A\cos\alpha + 2v_B\cos\beta)$	A1	
	$\frac{1}{8}\times(3u\cos\theta + u\cos\theta) = v_B\cos\beta - v_A\cos\alpha$	M1	Impact law
	$\left(\frac{u}{2}\cos\theta = v_B\cos\beta - v_A\cos\alpha\right)$	A 1	
	$\frac{u}{2}\cos\theta = v_B\cos\beta$, $0 = v_A\cos\alpha\,(\Rightarrow \sin\alpha = 1)$	DM1	Simultaneous equations
		A1	
	$v_A\sin\alpha = v_A = 2v_B = 3u\sin\theta$	M1	Use $v_A = 2v_B$ to find β
	$v_B\sin\beta = u\sin\theta \Rightarrow \frac{3u\sin\theta}{2}\sin\beta = u\sin\theta$	A1	Equation without v_A and v_B
	$\sin\beta = \frac{2}{3}$	A1	
	$2v_B = 3u\sin\theta$ & $\frac{u}{2}\cos\theta = v_B\cos\beta$ $\Rightarrow 6\tan\theta = \frac{2}{\cos\beta}\left(= 2\times\frac{3}{\sqrt{5}}\right)$	M1	Solve for θ
	$\tan\theta = \frac{1}{\sqrt{5}}$, $\theta = 24.1(^\circ)$ (0.421 radians)	A1	
		[12]	

Question Number	Scheme	Marks	Notes
4a	Equation of motion: $900a = \frac{22500}{v} - 25v$	M1	Requires all three terms. Condone sign errors
		A1	Correct unsimplified equation
	$a = \frac{\frac{22500}{v} - 25v}{900} = \frac{900 - v^2}{36v}$	A1 [3]	Obtain ****Given answer**** with no errors seen
4b	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{900 - v^2}{36v}$	B1	Differential equation in v and t
	$\int \frac{36v}{900 - v^2}\mathrm{d}v = \int 1\mathrm{d}t$,	M1	Separate & integrate to obtain a solution involving a log. function
	$t = -18\ln\left(900 - v^2\right)(+C)$	A1	
	: $T = -18\ln 500 + 18\ln 800 = 18\ln\frac{8}{5}$	DM1	Use limits correctly Dependent on previous M1
		A1	Obtain ****Given answer**** with no errors seen
		[5]	
4c	$\frac{900 - v^2}{36v} = v\frac{\mathrm{d}v}{\mathrm{d}x}$	B1	Differential equation in v and x
	$\int \frac{v^2}{900 - v^2}\mathrm{d}v = \int \frac{1}{36}\mathrm{d}x$	M1	Separate variables
	$= \int \frac{900}{900 - v^2} - 1\mathrm{d}v = \left(\int \frac{900}{60}\left(\frac{1}{30 - v} + \frac{1}{30 + v}\right) - 1\mathrm{d}v\right)$	M1	Split to the form $\frac{A}{900 - v^2} + B$ and integrate
	$15\ln\left\lvert\frac{30 + v}{30 - v}\right\rvert - v = \frac{x}{36}(+C)$	A1	
	$15\ln\left(\frac{50}{10} \times \frac{20}{40}\right) - (20 - 10) = \frac{x}{36}$	M1	Use limits and solve for x
	$x = 135$ (m) $\quad (540\ln 2.5 - 360)$	A1	Accept exact answer of the form $a\ln b - c$
		[6]	
		(14)	

Question Number	Scheme	Marks	Notes
5a			Extension in AP: $2+x$, Extension in BP: $3+\frac{1}{4}\sin 2t-x-1$
	$T_1=\frac{12(2+x)}{1}$	B1	Force towards A
	$T_2=12\left(2+\frac{1}{4}\sin 2t-x\right)$	B1	Force towards B
	$1.5\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=T_2-T_1\left(=3\sin 2t-24x\right)$	M1	Form equation of motion of P. Requires derivative and both tensions, but condone sign errors. Allow with a for $\ddot{x}$
		A1	Correct equation in x and t
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+16x=2\sin 2t$	A1	Obtain *****given answer***** with no errors seen.
		[5]	
5b	$t=0, x=0 \quad \Rightarrow C=0$	B1	
	$t=0, \dot{x}=0=\left(-4C\sin 4t\right)+4D\cos 4t+\frac{1}{3}\cos 2t$	M1	SR: M1A1 is available if C is not found or C is incorrect.
	$D=-\frac{1}{12}$	A1	
	$\dot{x}=0 \Rightarrow \cos 4t=\cos 2t$	M1	At rest: set their $\dot{x}=0$
	$2\cos^2 2t-1=\cos 2t$		
	$\cos 2t=1,-\frac{1}{2} \quad 2t=\frac{2\pi}{3}, \quad t=\frac{\pi}{3} \quad (1.05)$	A1	Not $\frac{1}{2}\cos^{-1}\left(\frac{-1}{2}\right)$?
		[5]	

Question Number	Scheme	Marks	Notes
6a			
	GPE of the ring: $-2mgr\cos 2\theta$	B1	Allow with $+2kmgr$
	GPE of suspended particles: $-\sqrt{6}mg\left(L_1-a\right)-\sqrt{6}mg\left(L_2-b\right)$	M1	Expression of the correct structure involving their L_1, L_2, a and b
	$a=2r\sin(45-\theta)=\dfrac{2r}{\sqrt{2}}\left(\cos\theta-\sin\theta\right)$	A1	Correct expression for BR in terms of r and θ Any equivalent e.g. $r\sqrt{2\left(1-\sin 2\theta\right)}$
	$b=2r\cos\left(45-\theta\right)=\dfrac{2r}{\sqrt{2}}\left(\cos\theta+\sin\theta\right)$	A1	Correct expression for AR in terms of r and θ Any equivalent e.g. $r\sqrt{2\left(1+\sin 2\theta\right)}$
	GPE of system: $-\sqrt{6}mg\left(L_1-a\right)-\sqrt{6}mg\left(L_2-b\right)-2mgr\cos 2\theta$	DM1	Add the three components. Dependent on the previous M
	$=2\times\dfrac{2r}{\sqrt{2}}\cos\theta\times\sqrt{6}mg-2mgr\cos 2\theta$ + constant		
	$=2mgr\left(2\sqrt{3}\cos\theta-\cos 2\theta\right)+$ constant	A1	Simplify to the **given answer**
		[6]	

Question Number	Scheme	Marks	Notes
6b	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = -4\sqrt{3}\ mgr\sin\theta + 4mgr\sin 2\theta$	M1	Differentiate
	In equilibrium: $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0 = 4mgr\sin\theta\left(-\sqrt{3} + 2\cos\theta\right)$	M1	Set $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ and solve for θ
	$\theta = \pm\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pm\frac{\pi}{6}(= \pm 0.52)$	A1	
	or $\theta = 0$	B1	
		[4]	
6c	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -4\sqrt{3}mgr\cos\theta + 8mgr\left(\cos^2\theta - \sin^2\theta\right)$	M1	Second derivative - needs to be the full expression.
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mgr\left(-4\sqrt{3}\times\frac{\sqrt{3}}{2} + 8\left(\frac{3}{4} - \frac{1}{4}\right)\right) = -2mgr < 0$	DM1	Substitute $\theta = \frac{\pi}{6}$ Dependent on the previous M1
	So equilibrium is unstable	A1	No errors seen
		[3]	
		(13)	
	Accept equivalent methods for determining max/min.		

Question Number	Scheme	Marks	Notes
7a	Resolve parallel to barrier - condone sin/cos confusion	M1	
	$u\cos 60 = v\cos\theta$	A1	
	Resolve perpendicular to the barrier - condone consistent sin/cos confusion. Use *e* correctly	M1	
	$eu\sin 60 = v\sin\theta$	A1	
	$v^2 = u^2\cos^2 60 + e^2u^2\sin^2 60 = \frac{u^2}{4} + \frac{3u^2}{16} = \frac{7u^2}{16}$	M1	Eliminate θ and solve for v.
	$v = \frac{\sqrt{7}}{4}u$	A1	Obtain given answer correctly with no errors seen
		[6]	
7b	Angle of approach with BC = 19.1°	B1	
	$v\cos 19.1 = w\cos\phi$	M1	Components parallel to BC condone sin/cos confusion
	$\frac{1}{2}v\sin 19.1 = w\sin\phi$	M1	Components perpendicular to BC condone consistent sin/cos confusion Use e correctly
		A1	Equations correct for their 19.1
	Form equation in v and ϕ	M1	Square and add or divide to find $\tan\phi$
	$w^2 = v^2\left(\frac{1}{4}\sin^2 19.1 + \cos^2 19.1\right)$	A1	$(\phi = 9.83°)$
	$0.634u$	A1	

Question Number	Scheme	Marks	Notes
7balt	$\tan\theta = \frac{1}{2}\tan 60$	B1	
	$\tan\alpha = \frac{1}{2}\tan(60-\theta) \quad \left(= \frac{1}{2}\left(\frac{\sqrt{3}-\frac{1}{2}\sqrt{3}}{1+\sqrt{3}.\frac{1}{2}\sqrt{3}}\right) = \frac{\sqrt{3}}{10}\right)$	M1	
		A1	
	$v\cos(60-\theta) = w\cos\alpha$	M1	
	$v\left(\frac{1}{2}.\frac{2}{\sqrt{7}} + \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{\sqrt{7}}\right) = w\frac{10}{\sqrt{103}}\left(= v\frac{5}{2\sqrt{7}}\right)$	M1	
		A1	
	$w = \frac{\sqrt{103}}{4\sqrt{7}}v = \frac{\sqrt{103}}{4\sqrt{7}}.\frac{\sqrt{7}}{4}u = \frac{\sqrt{103}}{16}u \qquad (0.634u)$	A1	
		[7]	